

Vidya Vikas Mandal's

Std : XII Ramacrisna Madeva Salgaocar Higher Secondary School Dur: 3 hr

Date : 24/01/2024

Margao – Goa

Marks : 80

Preliminary Examination

Subject : MATHEMATICS AND STATISTICS

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1. All questions are compulsory.
  2. The question paper consists of 36 questions.
  3. Question number 1 to 8 are multiple choice type question of one mark each.
  4. Question number 9 to 16 are very short answer type question of one mark each.
  5. Question numbers 17 to 22 are short answer type -I question of two marks each.
  6. Question numbers 23 to 28 are short answer type -II question of three marks each.
  7. Question numbers 29 to 34 are Long answer type -I question of four marks each.
  8. Question numbers 35 to 36 are Long answer type -I question of five marks each.
  9. There is no overall choice in the paper. However internal choice is provided in 2 question of 4 marks and in 2 question of 5 marks.
  10. Use of calculators is not permitted.
  11. Log tables will be supplied on request.
  12. Graph should be drawn on the answer paper only.
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1. If  $A = [a_{ij}]$  be an  $m \times n$  matrix, then the matrix obtained by interchanging the rows and columns of A is called - - - - .  
(A) Transpose of A  
(B) Adjoint of A  
(C) Inverse of A  
(D) None of the above

2. If  $x^2 + y^2 = 4$ , then  $\frac{dy}{dx} =$  -----.

(A)  $\frac{x}{y}$

(B)  $\frac{y}{x}$

(C)  $-\frac{x}{y}$

(D)  $\frac{-y}{2}$

3. Let  $f: Z \rightarrow Z$  be defined by  $f(x) = 3x^2$ , then  $f$  is -----.

(A) Injective but not surjective

(B) Surjective but not Injective

(C) Injective and Surjective

(D) Neither Injective nor Surjective.

4.  $\int \frac{\cos x \, dx}{2+5 \sin x} =$  -----.

(A)  $\log |\cos x| + c$

(B)  $\log |\sin x| + c$

(C)  $\frac{1}{5} \log |2 + 5 \sin x| + c$

(D)  $5 \log |2 + 5 \sin x| + c$

5. If  $A$  and  $B$  are two independent events with  $P(A) = \frac{2}{11}$  and  $P(B) = \frac{3}{11}$ ,

then  $P(A \cap B) =$  -----.

(A)  $\frac{2}{11}$

(B)  $\frac{3}{11}$

(C)  $\frac{6}{11}$

(D)  $\frac{8}{11}$

6. If the banker's discount on a bill of Rs 12,000 due in 8 months is Rs 480, then the rate of interest is ----

(A) 6%

(B) 8%

(C) 10%

(D) 12%

7. The order and degree of the differential equation

$$\frac{d^3y}{dx^3} + 4 \left[ \left( \frac{dy}{dx} \right)^3 + y^2 \right] = 0 \text{ is}$$

- (A) Order = 3 Degree = 1
- (B) Order = 2 Degree = 1
- (C) Order = 1 Degree = 1
- (D) Order = 1 Degree = N.D

8. At the break-even point -----.

- (A) Profit = Revenue function
- (B) Revenue Function = Cost function.
- (C) Average cost = Marginal cost
- (D) Profit = Cost Function

9. Define Ordinary Annuity.

10. Define Bill of Exchange after sight.

11. The probability that a person ordered a vanilla ice cream is 0.2 and the probability that he ordered a sundae is 0.3. The probability that he will order a sundae, given that he has already ordered a vanilla ice cream is 0.6. Find the probability that he will order both vanilla icecream and sundae.

12. The cost function of a firm is given by  $C = 7x^3 - 4x + 24$ . Find the marginal cost, when the output,  $x = 4$ .

13. If  $*$  is a binary operation on  $Z$  defined by  $a * b = 3a - b$ .

Find  $(2 * 3) * 1$ .

14. Define Gaining Ratio.

15. Differentiate  $y = \log(x^2 + 1)^5$  w.r.t  $x$ .

16. If  $P(A') = \frac{5}{8}$ ,  $P(B) = \frac{1}{2}$  and  $P(A \cap B) = \frac{1}{4}$ . Find  $P(B/A)$ .

17. Evaluate  $\int \sqrt{x^2 + 4x + 5} dx$ .

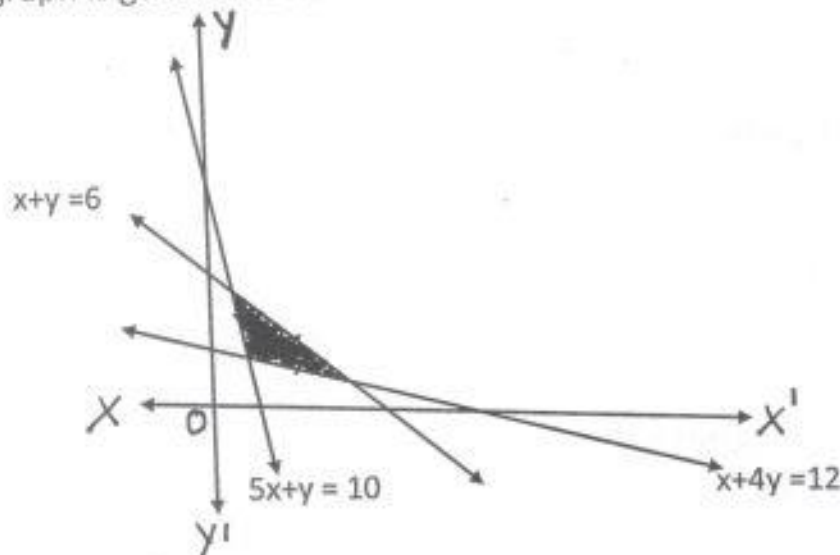
18. Form the differential equation of  $y^2 = 16a(x-b)$ , by eliminating the arbitrary constants  $a$  and  $b$ .

19. If  $x^y = \sin x$ , then find  $\frac{dy}{dx}$ .

20. Find  $x$ , if  $[-1 \ 2 \ 3] \begin{bmatrix} 2 & 4 & 1 \\ 0 & 3 & 5 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [18]$ .

21. If  $f(x) = 6 - 4x$  and  $g(x) = x^2$ . Find :  $f \circ g(x)$  and  $g \circ f(x)$ .

22. Write the constraints of the linear programming problem whose graph is given below.



23. Evaluate  $\int_0^1 x^2 \sqrt{1-x} dx$ .

24. Express the matrix  $A = \begin{bmatrix} 7 & 2 & 5 \\ 2 & 2 & 1 \\ 3 & 8 & 10 \end{bmatrix}$  as a sum of symmetric and skew-symmetric matrix.

25. By using properties of determinant, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

26. If  $B = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ . Find  $B^{-1}$ , using adjoint method.

27. Find the particular solution of the differential equation  $\frac{dy}{dx} = y \tan x$ , when  $x=0$  and  $y=1$ .

28. A person uses his car 30% of the time, walks 30% of the time and rides the bus 40% of the time, as he goes to work. He is late, 10% of the time when he walks, 3% of the time when he drives and 7% of the time when he take the bus. What is the probability he took the bus, if he was late?

29. Solve the following linear programming problem graphically,

$$\text{Minimise } Z = 3x + 5y$$

$$\text{subject to } x + 2y \leq 2000$$

$$x + y \leq 1500$$

$$y \leq 600$$

$$x \geq 0, y \geq 0$$

30. If  $f$  is continuous at  $x = 0$ , where

$$f(x) = \frac{\log(1+Ax)}{x}, \quad x < 0$$

$$= 5, \quad x = 0$$

$$= \frac{\sin Bx}{x} - 3, \quad x > 0$$

Find values of  $A$  and  $B$ .

31. Evaluate  $\int \frac{x^2 - 2x - 4}{(x-1)(x+3)(x-2)} dx$

OR

Evaluate  $\int \frac{x+5}{3x^2-2x-1} dx$

32. If  $y = \log(\log x)$ , prove that  $x \frac{d^2y}{dx^2} + \frac{dy}{dx} + x \left(\frac{dy}{dx}\right)^2 = 0$

33. If the bankers gain on a bill due 4 months hence at 6% per annum is Rs 800. Find the amount of the bill. Also find the true discount on it.

OR

A bill for Rs 21,900 drawn on July 10, for six months was discounted for Rs 21,720 at 5% per annum. On what date was the bill discounted?

34. The demand function for a commodity is given by  $x = 100 - 3p$ , where  $x$  is the number of units of the commodity produced and  $p$  is the price per unit. At what value of  $x$  will there be maximum revenue? What is the maximum revenue?



35. Priti, Niki and Nishu invested Rs 50,000, Rs 1,00,000 and Rs 5,00,000 respectively in a partnership firm. Each partner is to receive 5% interest on the money invested. Also, Priti and Niki are paid Rs 5000 per month each, for the first six months as salary. After the end of the first year the profit of the firm amounts to Rs 2,22,500. Find the share of profit received by each partner, if it is distributed in the ratio of their investments.

OR

A, B and C Form a partnership and contribute Rs5,00,000, Rs 4,00,000 and Rs 3,00,000 respectively towards capital. They agree to divide the annual profit in proportion to the capital employed and to the time it is in use. After 5 months, A withdraws Rs 50,000 and B adds Rs 50,000. At the end of the year, the profit is Rs 1,22,400. How should this be shared ?

36. Mr Rao bought a car paying Rs. 3,00,000 down payment and promising to pay Rs 6000 at the end of each quarter for next 5 years. If the car company charges interest 12% per annum compounded quarterly. Find the cash price of the car. ( Use log table).

OR

Ajit opens a recurring deposit account in a bank, which pays 6% per annum compounded monthly. Find how much money should be deposited by Ajit in the bank at the beginning of each month to accumulate Rs 2,63,000 in 4 years . ( Use log table).

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